A SIMPLE TECHNIQUE TO DETERMINE CALIBRATION PARAMETERS FOR COPLANAR CAMERA CALIBRATION

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ABSTRACT

Coplanar camera calibration is the process of determining the extrinsic and intrinsic parameters of camera, where a set of image and two-dimensional (2D) control points are known. This paper presents a simple and efficient technique to compute the calibration parameters for coplanar calibration. Computing of the calibration parameters is directly performed on computer image array in the frame buffer. Thereby, the image center can simultaneously computed together with other parameters. However, the parameters can be solved using available three orthogonality constraints, where the scale factor is assumed as one and the lens distortion is not considered. The proposed algorithm is tested with noise-free synthetic image and synthetic image corrupted with known noise. The results show that the proposed technique is accurate and robust to noise.

1. INTRODUCTION

Camera calibration is a necessary step in computer vision and robot vision for three-dimensional (3D) object measurement. It is the process to determine the camera intrinsic parameters such as focal length, scale factor, image center, lens distortion, and camera extrinsic parameters such as the 3D position and the orientation of camera which relate to certain world coordinate system.

According to the geometry of calibration pattern, there are two types of calibration, non-coplanar and coplanar camera calibrations. The former is calibrated by observing the control points of calibration pattern in the known 3D space with good precision. The latter is performed by observing the control points lying on 2D plane. Most of the camera calibration works have emphasized on non-coplanar calibration rather than coplanar case because non-coplanar calibration is easy and the number of usable orthogonality constraints is sufficient. However, there is several industrial and military applications required coplanar calibration.

Tsai [1] designed radial alignment constraint (RAC) for camera calibration in both coplanar and non-coplanar calibrations. Most parameters are computed in closed form by linear method, while some intrinsic parameters such as camera focal length, depth component of translation vector, and radial distortion coefficients are computed using standard nonlinear minimization scheme (steepest descent). However, the computed parameters use relationship between the control points of the world coordinate and their corresponding image points on image plane, where the image center and scale factor are assumed previously known. However, this is not a flexible method to compute and obtain the accurate parameters. Therefore, Lenz and Tsai [2] presented the nonlinear method for solving image center and scale factor by minimizing the RAC residual error, which the accuracy of the calibration parameters influenced from image center and scale factor is increased.

Chatterjee et al [3] proposed algorithms for the coplanar camera calibration which calibration parameters are computed by both linear and nonlinear optimization methods. In linear case, the non-coplanar algorithms of Ganapathy [4], Grosky and Tamiburino [5], and Chatterjee et al [6] are extended to the coplanar case. From these algorithms, some of the calculated scale factors and the entire image center are not simultaneously computed. Thereby, the computed parameters are lack of accuracy. This caused the error of the extrinsic parameters and 3D measurement

In this proposed technique, the computing of calibration parameters is directly performed on the computer image array in frame buffer, and the image center is simultaneously computed with other parameters using linear method by assuming that the scale factor is one and lens distortion is not considered. This proposed technique aims to develop a simple and efficient algorithm for coplanar case to obtain more accurate results than those of Tsai method and Grosky method extended by Chatterjee et al. For lens distortion correction, this proposed technique to nonlinear optimization method using the computed parameters from linear method as initial set could also be extended.

In Section 2, the camera model that represents the

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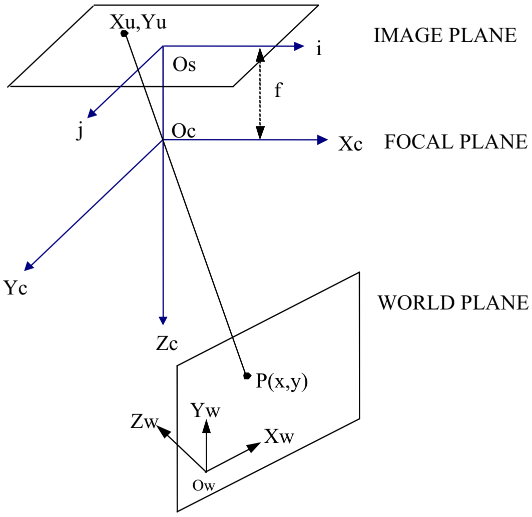

Fig. 1 Mapping control points to image plane

relationship between computer image coordinate and 2D control points will be described. In Sections 3, the camera calibration using linear method is solved. Finally, the experimental results with synthetic image and the comparison of the results to those of Tsai method and Grosky method are shown in Section 4.

2. CAMERA MODEL

Calibration parameters consist of extrinsic and intrinsic parameters. The extrinsic parameters consist of 3x3 rotation matrix R, which defines camera orientation, and 3x1 translation vector *t*, which defines 3D position of camera center. The intrinsic parameters consist of the effective focal length (f) of camera, center of image array (C_x, C_y), aspect ratio or scale factor (s) of the image array, and lens distortion.

Fig.1 illustrates the basic geometry of camera model, the origin of world coordinate system (O_w) lying on a calibration pattern is assumed. Thereby, the control points of calibration pattern lie on 2D plane (X_w, Y_w). 3D camera coordinate system (X_c, Y_c, Z_c) with optical center at O_c point and Z_c axis is the same as the optical axis. Transformation from the control points of 2D plane to camera coordinate system for coplanar calibration case ($Z_w = 0$) can be shown as

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} R \mid t \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix} \tag{1}$$

where R = $\begin{bmatrix} r_1 & r_2 \\ r_4 & r_5 \\ r_7 & r_8 \end{bmatrix}$

is 3x2 rotation matrix that defines camera orientation. *t* is 3x1 translation vector that defines the position of camera center.

2D image plane is placed behind the focal plane with (i, j) axes aligned with (X_c, Y_c) respectively. The effective focal length (f) is the distance between the image plane and the optical center. Considering the pinhole camera model, the relationship between the control points of the camera coordinate and their corresponding points on image plane is given by

$$X_u = -f\frac{X_c}{Z_c}$$
$$Y_u = -f\frac{Y_c}{Z_c} \tag{2}$$

or to be expressed in the matrix form as

$$\begin{bmatrix} \alpha X_u \\ \alpha Y_u \\ \alpha \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} \tag{3}$$

where (X_u, Y_u) are ideal undistorted image points on image plane and α is a non-zero scale factor.

Transformation from image plane to computer image array in the form of row and column in the frame buffer can be expressed by

$$X_u = n_i s^{-1}(X_f - C_x)$$
$$Y_u = n_j(Y_f - C_y) \tag{4}$$

where (X_f, Y_f) is an image coordinate (pixel) in the frame buffer. (n_i, n_j) is the distance between sensors in i and j directions, previously known from manufacturing's data. Here, the relationship of image coordinates in the frame buffer and image points on the image plane in the matrix form can be expressed as

$$\begin{bmatrix} \alpha X_f \\ \alpha Y_f \\ \alpha \end{bmatrix} = \begin{bmatrix} n_i^{-1}s & 0 & C_x \\ 0 & n_j^{-1} & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha X_u \\ \alpha Y_u \\ \alpha \end{bmatrix} \tag{5}$$

Finally, the relationship between the control points in 2D plane and their corresponding coordinates in the frame buffer can be written by

$$\begin{bmatrix} \alpha X_f \\ \alpha Y_f \\ \alpha \end{bmatrix} = \begin{bmatrix} -f_x r_1 + C_x r_7 & -f_x r_2 + C_x r_8 & -f_x t_1 + C_x t_3 \\ -f_y r_4 + C_y r_7 & -f_y r_5 + C_y r_8 & -f_y t_2 + C_y t_3 \\ r_7 & r_8 & t_3 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix} \tag{6}$$

However, the results of imperfections in the design and assembly of lens and the projection of the control points from 2D plane to image plane are not accurate. Types of lens distortions commonly seen are radial and tangential. Let (X_d, Y_d) be the actual distorted image coordinate on image plane, which causes from the lens distortion. The ideal undistorted image coordinate (X_u, Y_u) related to the actual distorted image coordinate can be expressed by

$$X_u = X_d + k_1 X_d r_d^2 + k_2 X_d r_d^4 + p_1\left(r_d^2 + 2X_d^2\right) + 2p_2 X_d Y_d$$
$$Y_u = Y_d + k_1 Y_d r_d^2 + k_2 Y_d r_d^4 + 2p_1 X_d Y_d + p_2\left(r_d^2 + 2Y_d^2\right) \tag{7}$$

where k_1, k_2 are first and second order of radial distortion coefficients respectively. p_1 and p_2 are tangential distortion coefficients, and $r_d^2 = X_d^2 + Y_d^2$.

3. SOLVING CAMERA CALIBRATION

This section describes the computing of calibration parameters directly performed on image coordinate in frame buffer by using linear method. The image center can be computed together with other parameters. Rigid body transformation from world coordinate to frame buffer in equation (6) can be rewritten in form of two collinearity equations:

$$X_f = \frac{(-sfr_1 + C_x r_7)X_w + (-sfr_2 + C_x r_8)Y_w + (-sft_1 + C_x t_3)}{r_7 X_w + r_8 Y_w + t_3}$$
$$Y_f = \frac{(-fr_4 + C_y r_7)X_w + (-fr_5 + C_y r_8)Y_w + (-ft_2 + C_y t_3)}{r_7 X_w + r_8 Y_w + t_3} \quad (8)$$

Since equation (8) has a nonlinear characteristic. Therefore, to easily obtain the estimate parameters by linear method, let assume that the number of row and column of the image center is equal ($C_x = C_y$), s=1, and lens distortion is ignored or corrected in advance.

The parameters to be calibrated consist of rotation matrix R with incomplete entries, translation vector *t*, focal length f, and image center C_x, C_y. Then equation (8) can be expressed in matrix form as

$$Ab = c \quad (9)$$

where

$$b^T = \begin{bmatrix} -\frac{fr_1 + C_x r_7}{t_3} & -\frac{fr_2 + C_x r_8}{t_3} & -\frac{fr_4 + C_x r_7}{t_3} & -\frac{fr_5 + C_x r_8}{t_3} & \frac{r_7}{t_3} & \frac{r_8}{t_3} \\ -\frac{ft_1 + C_x t_3}{t_3} & -\frac{ft_2 + C_x t_3}{t_3} & & & & \end{bmatrix}$$

$$A = \begin{bmatrix} X_{wi} & Y_{wi} & 0 & 0 & -X_{fi}X_{wi} & -X_{fi}Y_{wi} & 1 & 0 \\ 0 & 0 & X_{wi} & Y_{wi} & -Y_{fi}X_{wi} & -Y_{fi}Y_{wi} & 0 & 1 \end{bmatrix}.$$

With n control points, overdetermined system of linear equations can be established and solved for unknown vector *b* in linear least square system

$$\min\|Ab - c\| \quad (10)$$

When the solution of vector *b* is obtained, the rotation matrix R is rearranged or expressed in the form of

$$\begin{bmatrix} r_1 & r_2 \\ r_4 & r_5 \\ r_7 & r_8 \end{bmatrix} = \begin{bmatrix} -A(b_1 - C_x b_5) & -A(b_2 - C_x b_6) \\ -A(b_3 - C_x b_5) & -A(b_4 - C_x b_6) \\ b_5 t_3 & b_6 t_3 \end{bmatrix} \quad (11)$$

where $b^T = [b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ b_7\ b_8]$, and $A = \frac{t_3}{f}$.

Then the orthogonality constraints of rotation matrix are used and the calibration parameters can be solved as follows:

$$A = sign\sqrt{\frac{b_5^2 + b_6^2}{a^2 + b^2}} \quad (12)$$

$$C_x = \frac{b_1 b_5 + b_2 b_6 - A^2 ae}{b_5^2 + b_6^2 - A^2 ac} \quad (13)$$

$$t_3 = sign\sqrt{\frac{1 - A^4(e - cC_x)^2}{b_5^2 + b_6^2}} \quad (14)$$

$$f = \frac{t_3}{A} n_i \quad (15)$$

where $a = b_4 b_5 - b_3 b_6$, $b = b_1 b_6 - b_2 b_5$, $c = a + b$, $e = b_1 b_4 - b_2 b_3$, with negative or positive sign if the origin of world coordinate system O_w is in front or behind the camera. The remaining parameters can be solved as follows:

$$r_1 = -A(b_1 - C_x b_5),\ r_2 = -A(b_2 - C_x b_6),\ r_3 = At_3 a,$$
$$r_4 = -A(b_3 - C_x b_5),\ r_5 = -A(b_4 - C_x b_6),\ r_6 = At_3 b,$$
$$r_7 = b_5 t_3,\ r_8 = b_6 t_3,\ r_9 = A^2(e - cC_x),$$
$$t_1 = -A(b_7 - C_x),\ t_2 = -A(b_8 - C_x) \quad (16)$$

4. EXPERIMENTAL RESULTS

In order to obtain the experimental results, the synthetic data is generated, with a known set of extrinsic and intrinsic parameters. The 8x7 grid of control points at 30 mm. distance to simulate 2D calibration pattern is produced, that is related to the certain world coordinate system. Let assume that the image center (C_x, C_y) in frame buffer is at (256, 256) for 512x512 image size; scale factor s=1; focal length f=55 mm; both conversion parameter n_i=n_j=0.0367, camera center position is (-198.9551, -79.7462, -943.0575) in mm., rotation parameter in form of Euler angle α, ϕ and β are 0, 0.0873, and 0.0873 rad., respectively. With the given control points and camera parameters, the image coordinates X_f and Y_f in frame buffer are obtained from equation (8).

The proposed technique is experimented on two sets of data: 1) noise-free synthetic data and 2) synthetic data corrupted with known noise. In noise-free synthetic data case, the results of the calibration parameters computed by the proposed technique are compared the result with those obtained from Tsai method and Grosky method, and shown in Table 1. For the Tsai method, all parameters are only computed by linear least square method, so the both results can be directly compared. In another case, independent quantization noise with uniform distribution is added on the interval (-0.5, 0.5) pixel to the image coordinates. The 50 test data with different quantization noise for test algorithms are generated, and the calibration parameters from 50 data sets are computed by the proposed technique, Tsai method, and Grosky method to obtain 50 sets of each parameter.

Finally, their mean and standard deviation and relative error of each parameter can be computed from

$$\text{Relative error} = \frac{\|\text{true value - computed value}\|}{\|\text{true value}\|} \quad (17)$$

where the mean of 50 sets parameters is used.

The result of synthetic data corrupted with known noise is given in table 2.

The experimental results with noise-free synthetic data indicate that parameters computed by the proposed technique are more accurate than those obtained from Tsai method and Grosky method. Moreover, the image center can be computed. Comparing those methods to the proposed technique for solving calibration parameters with synthetic data corrupted with known noise, the Grosky method is more sensitive to noise. The Tsai method is less sensitive to noise compared to Grosky method, but the depth component of translation vector and focal length are more varying than other parameters. Therefore, Tsai used nonlinear minimization scheme in computing of t_3 and f in order to get the optimal solution. The proposed technique is generally more accurate and more robust to noise than other methods. Image center is rather accurate, although the parameters t_1 and t_2 of translation vector are more varying to noise.

5. CONCLUSION

In this paper, the simple and efficient technique in computing the calibration parameters of coplanar calibration directly performed on computer image array is presented. The image center is simultaneously computed with other parameters. The scale factor is set to one and lens distortion is ignored. The experimental results show that the parameters computed by the proposed technique are more accurate and robust to noise than those obtained from Tsai method and Grosky method. It is possible to use these parameters as an initial set for optimization scheme, where the number of iteration can be reduced and the optimal solution can be quickly reached.

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Table 1 Calibration parameter computed

	The proposed method	Tsai method	Grosky method
r1	0.99306	0.99306	0.99307
r2	0.086857	0.086857	0.08658
r3	-0.079284	-0.07284	-0.079232
r4	-0.079284	-0.079284	-0.079285
r5	0.9924	0.9924	0.9924
r6	0.094128	0.094128	0.094066
r7	0.086857	0.086857	0.086799
r8	-0.087189	-0.087189	-0.087131
r9	0.9924	0.9924	0.99241
t1	-198.9551	-198.9551	-198.9561
t2	-79.7462	-79.7462	-79.7466
t3	-943.0575	-943.0599	-942.4282
f	55.0000	55.0001	54.9630
s			1.0000
Cx	256.0000		
Cy	256.0000		

Table 2 Relative error and standard deviation of parameter estimates for synthetic data

	The proposed technique	Tsai method	Grosky method
r1	8.4587E-05 (0.000412)	0.000361 (0.001997)	0.003402 (0.023137)
r2	0.00028783 (0.000579)	0.00312 (0.0009)	0.055873 (0.006562)
r3	0.01117502 (0.004622)	0.122547 (0.028216)	0.052747 (0.076258)
r4	0.17940568 (0.100078)	0.007896 (0.002726)	0.127869 (0.008581)
r5	9.2705E-05 (0.000392)	6.05E-06 (0.000995)	0.002056 (0.04274)
r6	0.01489461 (0.004763)	0.015065 (0.012013)	0.027771 (0.090667)
r7	0.00896876 (0.004508)	0.112795 (0.028805)	0.342068 (0.146289)
r8	0.01450871 (0.004687)	0.006893 (0.010415)	0.020633 (0.09306)
r9	2.0153E-05 (0.000467)	0.000397 (0.002935)	0.003825 (0.059692)
t1	0.00293496 (4.817735)	0.000244 (0.325841)	0.003903 (3.993821)
t2	0.01794493 (14.99456)	0.001755 (0.528917)	0.009433 (3.740298)
t3	0.00879624 (39.50166)	0.090366 (203.7421)	0.002263 (657.095)
f	0.00921236 (2.328078)	0.088912 (11.74878)	4.78E-05 (36.76785)
s			0.000354 (0.028489)
Cx	0.00356987 (7.7248)		
Cy	0.00356987 (7.7248)		