Early Processing - Smoothing and Differentiating

Suppose:

$$f(x) : \text{image function}$$

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2} : \text{Gaussian smoothing function}$$

Then:

$$\begin{aligned} f * g &\Rightarrow \text{bandlimited approximation of image } f(x) \\ &\Rightarrow \text{suppress noise prior to differentiation} \\ \frac{d}{dt}(f * g) &\Rightarrow \text{edges in the image } f(x) \end{aligned}$$

Canny Edge Operator

$$\frac{d}{dt}(f * g) = f * \frac{d}{dt} g$$

Early Processing - Gaussian Operators

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$$

$$g'_\sigma(x) = \frac{-x}{\sqrt{2\pi}\sigma^3} e^{-x^2/2\sigma^2}$$

$$g''_\sigma(x) = \frac{1}{\sqrt{2\pi}} \left[\frac{x^2}{\sigma^5} - \frac{1}{\sigma^3} \right] e^{-x^2/2\sigma^2}$$

Center Frequency

$$\omega_0 = \frac{\sqrt{n}}{\sigma}$$

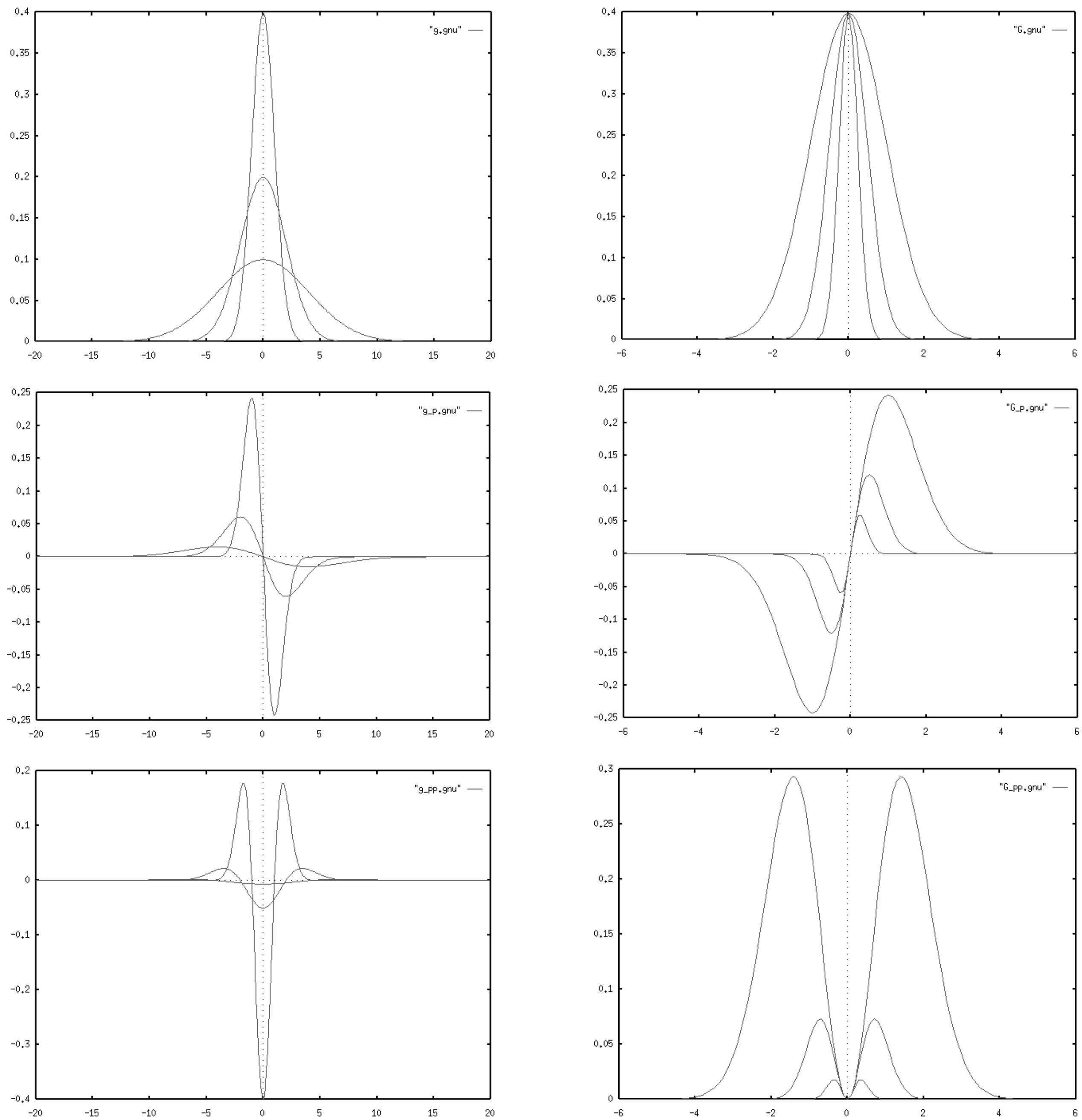
Equivalent Rectangular Bandwidth

$$n = 0 \qquad W = \frac{\sqrt{2\pi}}{4\sigma}$$

$$n = 1 \qquad W = \frac{e\sqrt{2\pi}}{8\sigma}$$

$$n = 2 \qquad W = \frac{3\sqrt{2\pi}e^2}{64\sigma}$$

Gaussian Operators in the Frequency Domain



Frei and Chen Texture Operators

"...probability of edge operators finding zero evidence of an edge anywhere in the image is quite small due to noise superimposed on low frequency variations."

⇒ sometimes, gradient magnitude thresholding does not work well

Consider a set of *basis* convolution operators:

k=0:
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

k=1,2:
$$\begin{bmatrix} -1 & -\sqrt{2} & -1 \\ & & \\ 1 & \sqrt{2} & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & & 1 \\ -\sqrt{2} & & \sqrt{2} \\ -1 & & 1 \end{bmatrix}$$

k=3,4:
$$\begin{bmatrix} & -1 & \sqrt{2} \\ 1 & & -1 \\ -\sqrt{2} & 1 & \end{bmatrix} \quad \begin{bmatrix} \sqrt{2} & -1 & \\ -1 & & 1 \\ & 1 & -\sqrt{2} \end{bmatrix}$$

k=5,6:
$$\begin{bmatrix} & 1 & \\ -1 & & 1 \\ & -1 & \end{bmatrix} \quad \begin{bmatrix} -1 & & 1 \\ & & \\ 1 & & -1 \end{bmatrix}$$

k=7,8:
$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

let:

$$g_k = f * h_k$$

Then:

$$\text{Edge Energy}(x) = \sum_{k=1}^{2} g_k^2(x)$$

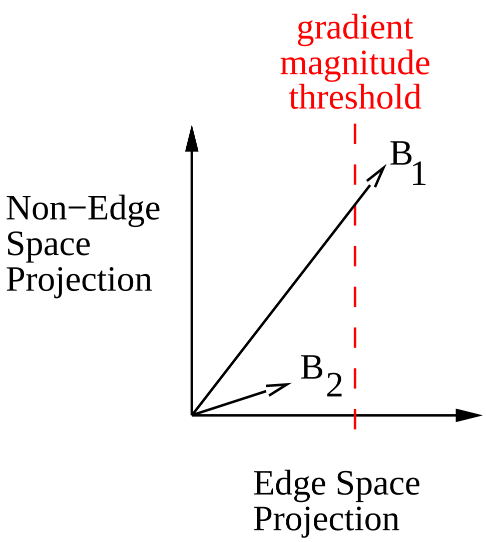
$$\text{Total Energy}(x) = \sum_{k=0}^{8} g_k^2(x)$$

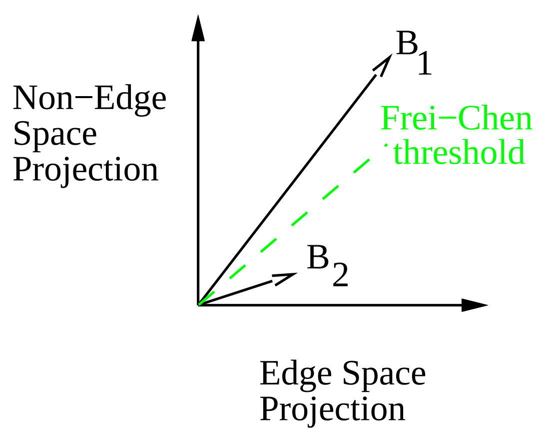
Frei and Chen Texture Operators - cont.

Define:

$$cos(\theta) = \left(\frac{EE}{TE}\right)^{1/2}$$

if $\theta > THRESHOLD$ report/label edge:



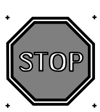


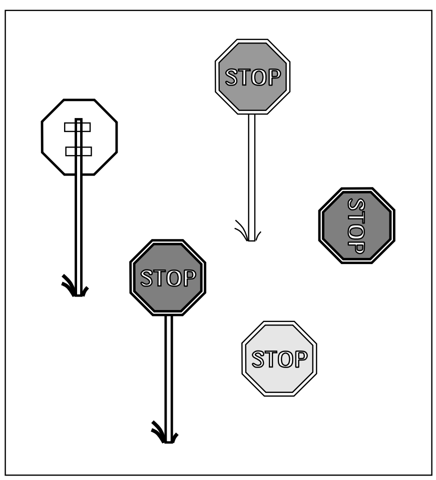
Early Processing - Cross Correlation

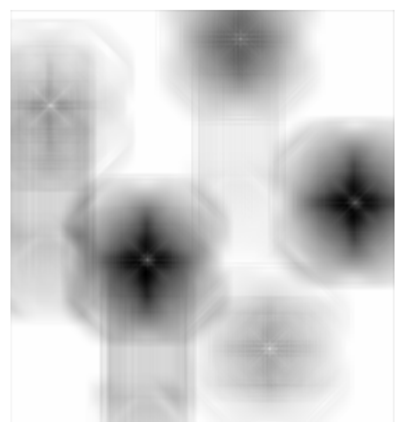
$$R_{gt} = \sum_{\alpha} \sum_{\beta} t(\alpha, \beta) g(x + \alpha, y + \beta).$$

maxima in R_{gt} are minima in:

$$\sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} [g(x+i, y+j) - t(\alpha, \beta)]^2$$







Normalized Cross Correlation

$$R(x,y) = \sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} \left[(g(x+i, y+j) - \hat{g}) \left(t(i+\alpha, j+\beta) - \hat{t} \right) \right] / (VW)$$

$$-1 \leq R(x,y) \leq +1$$

let: $M = (2\alpha + 1)$ and $N = (2\alpha + 1)$ represent the dimensions of the template, then

$$\hat{t} = \left[\sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} t(i+\alpha, j+\beta) \right] / (MN)$$

$$W = \left[\sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} (t(i+\alpha, j+\beta) - \hat{t})^2 \right]^{1/2} .$$

$$\hat{g} = \left[\sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} g(x+i, y+j) \right] / (MN)$$

$$V = \left[\sum_{i=-\alpha}^{\alpha} \sum_{j=-\beta}^{\beta} (g(x+i, y+j) - \hat{g})^2 \right]^{1/2} .$$

Normalized Cross Correlation