# IDENTIFICATION OF DYNAMIC MODEL WITH DEAD-ZONE FOR MOBILE ROBOT WITH DIFFERENTIAL DRIVE

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**Abstract**— The nonlinearities present in the actuators of wheeled mobile robots make them show undesirable characteristics and become more difficult to obtain a model of the robot. The dead-zone is one of the nonlinearities found very often in many robot actuators. Considering the dead-zone while modeling a mobile robot should lead to more precise models. This work proposes a identification of a robot model with dead-zone. The dead-zone is previously identified through experiments and it is used to identify a linear model to the robot using the mean least squares method. The results shows that the identified model with dead-zone represents better the robot than the identified model without dead-zone.

**Keywords**— Mobile Robot, Dead-Zone, Identification, Robot Modeling

## 1 Introduction

The mobile robots are robots that uses his actuators to produce his own movement through the environment. There are many kinds of mobile robots as well as models to describe them. These models are usually nonlinear, unless some simplification is made to generate a linear model. A linear model without simplifications was proposed and identified in Guerra et al. (2004). The linear model used allow the identification of the system through classical linear identification methods, like Least Mean Squares method.

The linear model proposed considers the robot as a linear system but, in fact, the robot itself has nonlinearities due to various phenomena of friction between mechanical parts. One of the most noticeable is the dead-zone that is the range of input signals that do not generate an output in the system. Take into account the dead-zone while modeling mobile robots should result in more realistic models.

In this work a identification of a mobile robot and his dead-zone is proposed. The robot is modeled like being composed of a nonlinear block, representing the dead-zone, connected in cascade with a linear block, representing the robot itself. First a phenomenological identification of the dead-zone is made and that information is used to identify the robot model, using the linear model proposed in Guerra et al. (2004).

The rest of the paper is organized as follows. Section 2 and section 3 describes the robot and its modeling. Section 4 presents the phenomenological identification of the dead-zone. Section 5 shows the obtained results. Finally, section 6 presents the conclusions and the future perspectives.

## 2 ROBOT MODELING

Fig. 1 presents a diagram of the kind of robot we consider in this work. The robot has two wheels, driven by two independent electric motors. The wheels are placed at each side of the robot, in such a position that their rotation axis are coincident. The robot configuration is represented by the position of the center of the axis between the two wheels in the Cartesian space ($x$ and $y$) and by its orientation $\theta$ (angle between the vector of the robot orientation and the abscissas axis).

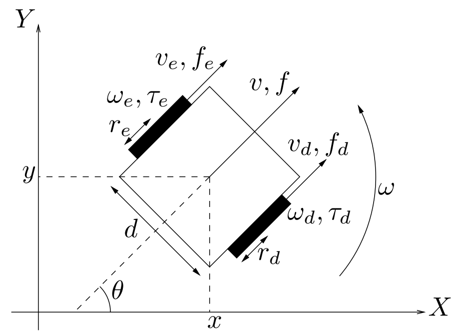


Figure 1: Mobile Robot

### 2.1 Cinematic Model

The cinematic model describes the relations between the derivatives of robot position and orientation and the robot linear and angular speeds, $v$ and $w$, without taking into account the causes of its movement. The cinematic model for the considered robot in this work is presented (1).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ w \end{bmatrix} \quad (1)$$

The equation (1) models the non-holonomic restrictions of the robot.

## 2.2 Dynamic Model

The dynamic model is derived from the physics laws that govern the several robot subsystems, including the actuator dynamics (electric and mechanical characteristics of the motors), friction and robot dynamics (movement equations). The derivation of this model for a small mobile robot was presented by (Vieira et al., 2001).

For most robots, the modelling process generates a second-order model expressed by:

$$\mathbf{Ku} = \mathbf{M}\dot{\mathbf{v}} + \mathbf{Bv} \tag{2}$$

where $\mathbf{v} = \begin{bmatrix} v & w \end{bmatrix}^T$ represents the robot linear and angular speeds, $\mathbf{u} = \begin{bmatrix} u_r & u_l \end{bmatrix}^T$ contains the input signals (usually armature tensions) applied to the right and left motors, $\mathbf{K}$ is the matrix which transforms the electrical signals $\mathbf{u}$ into forces to be generated by the robot wheels, $\mathbf{M}$ is the generalized inertia matrix and $\mathbf{B}$ is the generalized damping matrix, which includes terms of viscous friction and electric resistance.

## 2.3 Equivalent Linear Model

The complete robot model, obtained from the union of equations (1) and (2), can be represented by the following state equation:

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \multicolumn{2}{c}{-\mathbf{M^{-1}B}} & \vdots\ \mathbf{0} \\ \cdots & \cdots & \cdots \\ \cos(\theta) & 0 & \\ \sin(\theta) & 0 & \vdots\ \mathbf{0} \\ 0 & 1 & \end{bmatrix} \begin{bmatrix} v \\ \omega \\ x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{M^{-1}K} \\ \cdots \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} \tag{3}$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \\ x \\ y \\ \theta \end{bmatrix}$$

where the system output $\mathbf{y} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$ corresponds to the robot configuration.

To allow the application of linear discretization techniques, we will rewrite the system equations into a linear representation of the robot dynamic behavior. To attain this objective, we need to change the set of state variables: the robot's configuration, given by its position $x$ and $y$ and its orientation $\theta$, will be described in terms of the robot linear displacement $l$ and the robot orientation $\theta$.

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \dot{l} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \multicolumn{2}{c}{-\mathbf{M^{-1}B}} & \vdots\ \mathbf{0} \\ \cdots & \cdots & \cdots \\ 1 & 0 & \vdots\ \mathbf{0} \\ 0 & 1 & \end{bmatrix} \begin{bmatrix} v \\ \omega \\ l \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{M^{-1}K} \\ \cdots \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} \tag{4}$$

$$\mathbf{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \\ l \\ \theta \end{bmatrix}$$

The new state equation (4) has the same dynamic parameters than the original state equation (3) but a new system output $\mathbf{z} = \begin{bmatrix} l & \theta \end{bmatrix}^T$. We observe that the linear equivalent model was obtained without any simplification assumption. In this way, the linear model is an exact equivalent representation of the original non-linear representation in (3). However, the "$l$" variable is not measurable: this aspect will be explored later.

## 2.4 Model Discretization

To allow the application of the classical estimation techniques, it is usual deriving a discrete transfer function equivalent to the model in (4). The first step is the transformation of the state space form into a continuous transfer matrix:

$$\begin{bmatrix} L(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} U_r(s) \\ U_l(s) \end{bmatrix} \tag{5}$$

where each term $G_{ij}(s)$ has the following structure:

$$G_{ij}(s) = \frac{N_{ij}(s)}{D(s)} = \frac{\beta_{ij}(s-n_i)}{s(s-p_1)(s-p_2)}$$

Analysing the robot model, we see that the two poles of $G_{ij}(s)$ are one mainly related to linear characteristics of the robot and the other, to angular ones. Also, if the robot has only small asymmetries, there will be a zero at each $G_{ij}(s)$ roughly canceling the non-related pole. Thus, we can make the following simplification:

$$G_{ij}(s) = \frac{\beta_{ij}(s-n_i)}{s(s-p_1)(s-p_2)} \simeq \begin{cases} \frac{\beta_{ij}}{s(s-p_l)} & \text{if } i=1 \\ \frac{\beta_{ij}}{s(s-p_\theta)} & \text{if } i=2 \end{cases}$$

In the second step, we calculate the four discrete transfer functions $G_{ij}(z)$:

$$\begin{bmatrix} L(z) \\ \theta(z) \end{bmatrix} = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \cdot \begin{bmatrix} U_r(z) \\ U_l(z) \end{bmatrix} \tag{6}$$

where each term $G_{ij}(z)$ has the following structure:

$$G_{ij}(z) = \frac{N_{ij}(z)}{D_i(z)} = \frac{\delta_{ij}z - \epsilon_{ij}}{(z-1)(z-\alpha_i)}$$

To obtain a parametrization appropriated to the estimation, the transfer functions are converted to equivalent difference equations:

$$\begin{aligned}\Delta l_k =& \alpha_1 \Delta l_{k-1} \\ &+ \delta_{11} u_{r,k-1} - \epsilon_{11} u_{r,k-2} \\ &+ \delta_{12} u_{l,k-1} - \epsilon_{12} u_{l,k-2}\end{aligned} \quad (7)$$

$$\begin{aligned}\Delta \theta_k =& \alpha_2 \Delta \theta_{k-1} \\ &+ \delta_{21} u_{r,k-1} - \epsilon_{21} u_{r,k-2} \\ &+ \delta_{22} u_{l,k-1} + \epsilon_{22} u_{l,k-2}\end{aligned} \quad (8)$$

where $\Delta l_k = l_k - l_{k-1}$ is the linear distance traveled in a sampling period, $\Delta\theta_k = \theta_k - \theta_{k-1}$ is the angular increment in the robot's orientation in the same period, and $u_{r,k}$,$u_{l,k}$ are the plant input signals.

## 3 Traveled Distance Computation

The variable $\Delta l$ is not measurable. In spite of this, we can use heuristics to calculate a plausible value for this variable, $\widetilde{\Delta l}$. The methodology developed to calculate $\widetilde{\Delta l}$ consists of the following steps:

1. calculation of the direction of the movement, i.e. the sign of $\widetilde{\Delta l}$

2. calculation of the path length $|\widetilde{\Delta l}|$

### *3.1 Computation of the Movement Direction*

To obtain the sign of the traveled distance, i.e. knowing if the robot moved forward or backward, we calculate the value of ${}^0x_1$, the $x$ coordinate of the current configuration point $(x_1, y_1)$ calculated with respect to the reference frame attached to the previous robot position $(x_0, y_0)$, as indicated in the Fig. 2. If ${}^0x_1$ is positive, the robot will have moved forward and $\widetilde{\Delta l} > 0$. Otherwise, the robot will have moved backward and $\widetilde{\Delta l} < 0$.

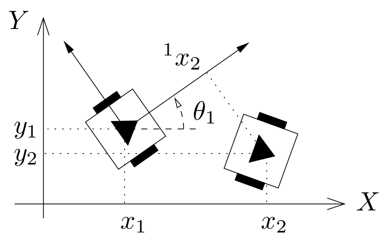


Figure 2: Diagram for calculating the sign of $\widetilde{\Delta l}$

A simple coordinate transformation allows the calculation of ${}^0x_1$:

$$\begin{aligned}{}^0x_1 = x_1 \cos\theta_0 - x_0 \cos\theta_0 + \\ + y_1 \sin\theta_0 - y_0 \sin\theta_0\end{aligned} \quad (9)$$

### *3.2 Path Length Computation*

It is possible to obtain with reasonable precision the robot configuration $(x, y, \theta)$ in two consecutive sampling instants, but we cannot be sure about the path traveled by the robot between the first and the second configuration. However, a estimated value of $|\widetilde{\Delta l}|$ can be calculated using a bezier curve of third degree (*Curves and Surfaces for CAGD: A Practical Guide*, 1996) between consecutive samples, where the initial orientation of the curve is equal to the robot orientation in the previous sample and the final orientation is equal to the actual robot orientation. Each control point of the curve is located in a straight line formed by the position and angle of the robot in each sample, with a distance of the robot position equal of a third of the euclidean distance between the samples, as shown in Fig. 3. The estimated value $|\widetilde{\Delta l}|$ can be achieved by numerical integration of the bezier curve. Note that the control points should agree with the movement direction of the robot (forward or backward).

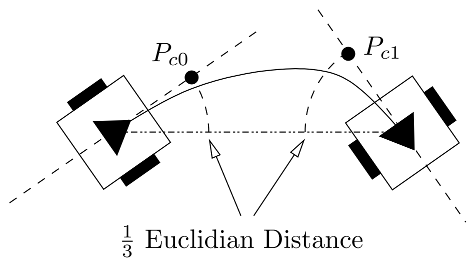


Figure 3: Diagram for $|\widetilde{\Delta l}|$ computation

## 4 Experimental Dead-Zone Identification

The model described above is linear, but the robot still have some nonlinearities that the model does not take into account. The dead-zone is a nonlinear phenomena caused by the friction present in the system, more specifically, the friction between the axis of the engines and their sockets, between the gears of the actuators and between the robot and the ground. The dead-zone is one of the most remarkable nonlinearities and his effect is more noticeable when the robot is excited with input signals with low amplitude.

The dead-zone can be modeled by a nonlinear input. A dead-zone with input $r(t)$ and output $u(t)$ is shown in the figure 4 and is described by equation 10 (Tao and Kokotovic, 1994). The figure 4 shows the input signal $r(t)$ being transformed in the signal $u(t)$ that is applied in the system. Note that $u(t)$ is virtual, ie, non-measurable.

$$u(t) = \begin{cases} m_r(r(t) - b_r) & \text{if } r(t) \geq b_r \\ 0 & \text{if } b_l < r(t) < b_r \\ m_l(r(t) - b_l) & \text{if } r(t) \leq b_l \end{cases} \quad (10)$$

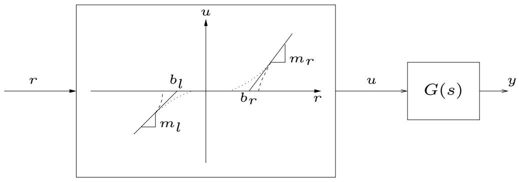


Figure 4: Dead-zone model

The equation 10 has four parameters, $b_r$,$b_l$,$m_r$ and $m_l$, meaning that the dead-zone can be asymmetric. Since the model has two input signals, there will be two dead-zones and then eight parameters to be estimated. Those parameters were experimentally estimated. The experiment consisted in vary one input signal trough all possible values while the other input was fixed to zero. In this configuration, the robot makes circular movement with its center located at the stationary wheel. The key idea is compare the non-constant input signal with the steady state angular velocity of the robot and observe for what input values the robot starts his movement and how the variation of angular velocity occurs. The figure 5 shows the results of the experiment described above.

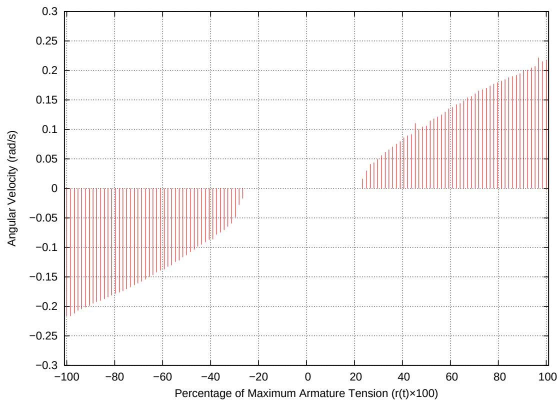


Figure 5: Result of experiment to estimate the dead-zone

The estimation of $b_r$ and $b_l$ is obtained direct by the graph being the minimal input value that generated angular velocity. Considering that when the maximum input value is applied the virtual value will be maximum and equal to the input value, $m_r$ and $m_l$ can be calculated respectively by equations 11 and 12. The experiment can be repeated swapping the input signals to estimate the parameters of the other dead-zone.

$$m_r = \frac{\max\left[r(t)\right]}{\max\left[r(t)\right] - b_r} \tag{11}$$

$$m_l = \frac{\min\left[r(t)\right]}{\min\left[r(t)\right] - b_l} \tag{12}$$

## 5 Results

This section presents the results of the identification of the mobile robot model using the experimentally identified dead-zone model. The data were obtained from a small mobile robot with two independent DC motors used in robot soccer competitions (Cerqueira et al., 2005). The input signals $e_r$ and $e_l$ are the armature voltages of the right and left DC motors, and the output $\mathbf{y} = [\begin{array}{ccc} x & y & \theta \end{array}]^T$ is measured with a global vision system(Aires et al., 2001).

The classical method of Recursive Least Mean Squares (Aguirre, 2007) was utilized for the estimation of the parameters of the model. The system was excited with pseudo-random input signals, with values between 1.0 and $-1.0$ (meaning 100% and $-100\%$ of the motor supply voltage). The non-measurable signal is calculated using the dead-zone model and is used to make the regressor matrix of the least squares method instead of using the original input signals.

The figures 6 and 7 shows respectively $\Delta l$ and $\Delta\theta$ obtained from the identified model with dead-zone using pseudo-random input signals and the measured values obtained by exciting the real robot with the same input signals.

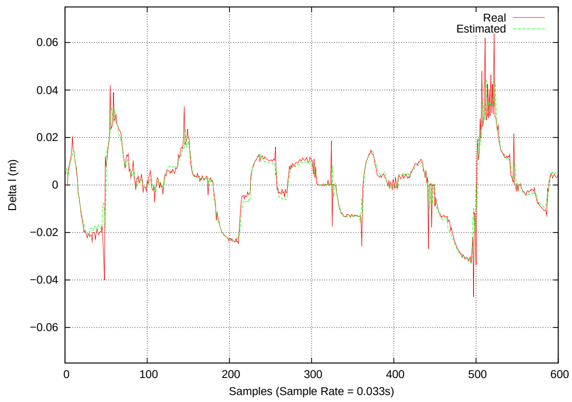


Figure 6: Comparison between calculated (“real”) $\Delta l$ and $\Delta l$ estimated by the model

## 6 Conclusion

The identified model with dead-zone shows a good approximation to the real system. When making a prior phenomenological identification of the dead-zone and using it to update the input signals, we let the system to be identified with less nonlinearities, causing the identified model to be more like the real system. Comparisons between that scheme and the direct identification method shows a slight improvement of the identified model due the addition of the dead-zone to the model.

One drawback of the method is that the dead-zone may be time-variable, due to the wastage of

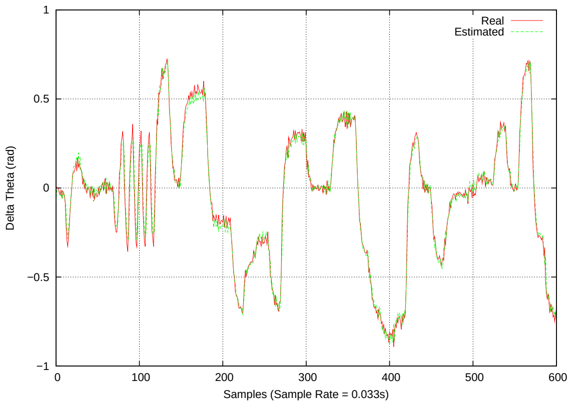


Figure 7: Comparison between measured ("real") $\Delta\theta$ and $\Delta\theta$ estimated by the model

mechanical parts or decrease of the power supply when the robot is powered by batteries. Nevertheless, in these cases the spin experiments and the identification can be remade to update the parameters of dead-zone and of the robot model.

The proposed approach can be improved by adapting existing techniques (Vörös, 2004) to also identify on line the dead zone parameters; the main difficulty in this case is to extend the monovariable existing proposals to this multi-variable context.

Future works will use the identified model to control the robot (Vieira et al., 2004), using techniques of adaptive control.

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